A note on Solow's early reaction to

Expectations-augmented Phillips curves

Michaël Assous[1]

(Université Lumière-Lyon2, Triangle)

Abstract

In three lectures given in Oxford in 1968-69, Robert Solow used a macro model developed in collaboration with Joseph Stiglitz to question Friedman's interpretation of the Phillips curve. In the specific case where the economy may either tend to demand- or supply-limited equilibria, he considered the possibility to define multiple non-

accelerating inflation rate of unemployment. By means of unedited correspondence and unpublished manuscripts from Solow's archives, the paper explores this modeling strategy with the aim to cast new light on how a prominent Keynesian in the late 1960s strived to challenge Friedman.

[1] CONTACT Michaël Assous ✉ michael.assous@univ-lyon2.fr ✉ Economie, Lumière University Lyon 2, 14 Avenue Berthelot, Lyon 69007, France. Sylvie Rivot and Robert Dimand and participants at the workshop Co-ordination in Economics Les Treilles, *May 29 –June 3 2017* provided *helpful and motivating comments on earlier drafts.*

1. Introduction

In 1958, Phillips (Phillips 1958, 283) advanced the hypothesis that changes in money wages had two causes: unemployment and its rate of change[2]. Along the same line, Solow and Samuelson (1960) conjectured that the tighter the labor and goods markets, the faster wages and prices would rise. In his 1967 presidential address to the American Economic Association Friedman transformed that line of argument meant to reflect market disequilibria into a description of equilibrium dynamics. As Solow later emphasized it: "after Milton's address, everybody treated [The Phillips curve] as an equilibrium matter" (Solow 2009, 77) with workers and firms assumed to permanently be on their labor demand and supply curves but with labor supply curves shifting in the short run due to mistaken price expectations[3]. In the long run, however, as soon as expectations of price inflation would adjust, workers would insist that wages increase at

---

[2] See also Lipsey (1960).

[3] This, of course, did not happen overnight. One must recognize Friedman (1968) was initially ambiguous. On the one hand, he considered unemployment as an excess supply on the labor market and thus implicitly relied on a disequilibrium interpretation of the Phillips curve. On the other hand, admitting that workers and firms remain on their labor and supply curves all along the adjustment process, his analysis seems to be based on an equilibrium interpretation. It is only in the mid 1970s that Friedman (1977) definitely opted for that second interpretation and eliminate any ambiguity. See Schwarzer (2013) for a detail account of Friedman's analysis.

the same rate as prices and labor supply curves would shift back to their initial position[4] while all possibility to permanently trade high inflation for lower unemployment would vanish.

These short- and long-run relations between inflation and unemployment were soon combined in a single "expectation-augmented Phillips curve": a short-run positively sloped curve associated with accelerating inflation and a long-run vertical curve consistent with stable inflation. On this basis, Friedman concluded that any equilibrium disturbed by demand policies would eventually return to its initial position with unemployment stabilizing at its "natural" level.

In three lectures given at the University of Manchester in March 1969 and published the year after under the title *Price expectations and the Behavior of the Price Level* (1970), Solow attempted to dismiss Friedman's analysis. Lectures 1 and 2 were empirical tries to show that the pass-through of 'expected' price inflation into actual inflation was not detectable at the "business time scale" while lecture 3 was an attempt to provide a new theoretical analysis of Phillips curves. That model, drawn from an article published in 1968 in the *Quarterly Journal of Economics* and written in

---

[4] Friedman thought that because consumption bundle of workers was made up of a whole multitude of items, a correct assessment of the real wage in terms of the price index consumption was likely to be far more difficult than calculation by firms of the real cost of labor to them, i.e. in terms of the price of the firm's own output. So, unlike firms, he claimed workers' labor supply decisions were based on incorrect information. Only at the "natural rate of unemployment" would expectations of workers and firms be consistent and fulfilled.

collaboration with Joseph Stiglitz, simultaneously brought supply- and demand-limited equilibria into the picture in a context in which both prices and wages were assumed to adjust - but not immediately - to disequilibria in goods and labor markets. Building on this property, Solow eventually set out the conditions to have multiple demand- and supply-limited short-run equilibria[5]. From a Keynesian perspective, this configuration was interesting. It means the possibility to define multiple non-accelerating inflation rate of unemployment and to offer a new understanding of the trade-off between inflation and unemployment.

The paper is organized in two parts. First, we show how Solow came with a particular understanding of Expectations-augmented Phillips curves based on his 1968 model. Second, we show how, by exploring the possibility of multiple equilibria, Solow eventually came to challenge Friedman's notion of natural rate of unemployment and its policy implications.

2. Phillips curves in disequilibrium

Solow-Stiglitz 1968 model is based on the notion that firms are either supply- or demand-constrained. Unlike fixed-price models later developed in the 1970s, it however incorporates price and wage equations by which nominal prices are assumed to respond

---

[5] Soon afterwards, the *Quarterly Journal of Economics* published a comment by Weintraub (1970) which did not much to highlight its originality. (see Solow 2003, 271). Later, though the paper proved to have much in common with path breaking contributions to disequilibrium theory - as evidence by its reprinting in the Korliras and Thorn 1979 book of macroeconomics reading - it kept attracting little attention.

to supply-demand tensions and changes in wages, while nominal wages are supposed to respond to unemployment and changes in prices. The analysis is limited to the short run in the Keynesian sense that the stock of capital goods is treated as constant: investment is only recognized as a determinant of aggregate demand.

Employment dynamics

On the supply side, the firm is assumed to hire extra labor until the marginal product of labor equals the real wage. Aggregate supply at any real wage is hence the output corresponding to the employment level at which the marginal product of labor equals the real wage and is, because of diminishing returns, a decreasing function $G(\cdot)$ of the real wage

$$Y^s = G(v), G' < 0 \tag{1}$$

where Y^s denotes aggregate supply and v the real wage.

The model approaches aggregate demand by the way of an IS curve. On the presumption that the marginal propensity to consume wage incomes, s_w, is higher than the marginal propensity to consume profit income, s_π, the aggregate demand function gives the sustainable real demand generated by any exogenous rate of investment I and real wage bill vN

$$Y^d = A + BvN \tag{2}$$

where A stands for I/s_π and B for $(s_\pi - s_w)/s_\pi$ [6].

---

[6] From equilibrium in the goods market $Y^d = I + (1 - s_w)vN + (1 - s_\pi)(Y^d - vN)$ which the IS equation is easily deduced: $Y^d = \frac{I}{s_\pi} + \frac{s_\pi - s_w}{s_\pi} vN$.

Investment spending is left exogenous so that the rate of interest rate plays no role. This may seem odd in a "theory in which money wages and prices play an important role." (Solow and Stiglitz 1968, 539) Though, the model was intended to be extended to include the monetary sector. In the meantime, Solow and Stiglitz suggested to assume that the monetary authorities were able to implement any policy likely to maintain a constant rate of interest.

The model postulates the short-side principle that employment adjusts toward the level corresponding to the smaller of aggregate demand and aggregate supply

$$\dot{N} = \theta\left(F^{-1}(min(Y^s, Y^d) - N)\right) \tag{3}$$

where $\dot{N}$ stands for dN/dt, $F^{-1}(\cdot)$ for the inverse function of the short-run production function $Y = F(N)$ and θ for a speed of adjustment.

When output is demand-limited, many firms would be willing and able to produce more at current prices and wages, but are deterred to do so by the "*force majeure* of effective demand"[7]. In that case, as long as s_π remains lower than s_w, employment appears as an increasing function of the real wage while the locus along which it is constant is positively sloped in the (N, v) plane. Conversely, when output is supply-limited, in the sense that at any higher output marginal cost would exceed marginal revenue, employment appears as a decreasing function of the real wage while the locus along which it is constant is now positively sloped.

---

[7] This means that despite the excess of price over marginal cost, all producers are not willing to produce more than what the market can bear. It is worth noting that this has nothing to do with markets imperfections in the goods market.

Let us now turn to the other side of the argument and the dynamics of the real wage in relation to employment.

Real wage dynamics

As for prices, the model is based on the assumption that its rate of change is simultaneously demand and cost determined "under some markup formula."[8] The price equation is hence assumed to have two explanatory variables: 1) the ratio between aggregate demand and aggregate supply and 2) the rate of change of the money wage (or the rate of change of unit labor cost, which comes to the same thing when the rate of productivity is given)

$$\frac{\dot{p}}{p} = g\left(\frac{Y^d}{Y^s}\right) + j\frac{\dot{w}}{w} \qquad (4)$$

Where $\dot{p}/p$ and $\dot{w}/w$ stand for the rates of change of the price level p and money wage w, $g(\cdot)$ is an increasing function of Y^d/Y^s and j is a coefficient comprised between zero and one. If it is equal to one, this means that firms are always able to pass on wage increase to their prices.

---

[8] The rate of change of price depends on the rate of change of labor cost per unit of output which itself depend on the rate of change of money wages and marginal product of labor. So, as long as the marginal product of labor and the markup set by firms are treated as constant, fluctuations in the rate of change of price is proportional to fluctuations in the money wage." (Solow and Stiglitz 1968, 544)

Money wage is treated symmetrically. The wage equation has: 1) the ratio of current employment to the supply of labor N^s- assumed exogenous - and 2) the rate of change of the price level.

$$\frac{\dot{w}}{w} = h\left(\frac{N}{N^s}\right) + k\frac{\dot{p}}{p} \tag{5}$$

where $h(\cdot)$ is an increasing function. The constant k is between zero and one. If it is equal to one, this means the wage bargain is struck in real terms and the rate of change of price is perfectly expected. Instead, a value of k close to zero means full money illusion.

Once solved simultaneously for the rate of change of the price level and money wage, the two "trade-off" equations give the dynamics of the real wage. Since Y^d is a function of both the real wage v and employment N while Y^s is a function of v alone, the dynamics of the real wage is given by

$$\frac{\dot{v}}{v} = h\left(\frac{N}{N^s}\right) - \frac{1-k}{1-j}g\left(\frac{A+BvN}{G(v)}\right) \tag{6}$$

or

$$\frac{\dot{v}}{v} = L(N) - C(N,v) \tag{6'}$$

with $L(N) = h\left(\frac{N}{N^s}\right)$ and $C(N,v) = \frac{1-k}{1-j}g\left(\frac{A+BvN}{G(v)}\right)$ whose derivatives L', C_v and C_N are all positive[9].

---

[9] Unfortunately, Solow and Stiglitz do not offer much insights on the reasons why coefficients j and k may change over time .

The slope of the locus along which v is constant in the (N, v) plane is given by $\frac{dN}{dv}\Big|_{\dot{v}=0} = \frac{C_v}{L' - C_N}$ whose sign is depending on the difference between L' and C_N. From the equation, v is an increasing function of employment if L' exceeds C_N, a decreasing function if the inequality is reversed.

The first case arises when an increase in real wage, through its positive impact on aggregate demand, takes an increase in money wages - allowed by a rise in employment - to stabilize the real wage at its higher level. This is more likely to happen the larger is coefficient k - the less there is money illusion in the labor market - and the smaller is constant B - the less aggregate demand is sensitive to the real wage. The argument is reversed when higher real wage, besides making money wages to rise, stimulates aggregate demand thereby making the price level to increase faster, so that it now takes a fall in employment to stabilize the real wage. According to Solow, this second case would refer to Kaldor's analysis of the dynamics of profit margin supposed to increase when employment is higher than its equilibrium level and to weaken when it falls below it. (see Solow 1970, 6)[10]

Except in the very special case that the constant-v curve intersects the constant-N curve right at its kink, the goods market is not cleared at "equilibrium" and there is either excess supply or excess demand of goods. As a result, according as the real wage

---

[10] Solow does not refer to any particular paper of Kaldor. But guesses can be made. It is very likely he thought of Kaldor 1955 paper in which the particular flexibility of prices relative to wages was brought up for the first time.

is an increasing or decreasing function of employment, at least two stable short-run equilibria can be defined.

Stability and multiplicity of Short run equilibria

The left picture represents a stable short-run supply-limited equilibrium at the intersection between the upward sloping $\dot{v} = 0$ curve and the $\dot{N} = 0$ curve on the branch where it is decreasing - for which output is supply determined[11]. In that situation, the price is equal to marginal cost and firms have no incentive to expand or contract output and employment. The price level may increase, but the money wage is just keeping pace, so that the real cost remained constant. Most importantly, whatever the parameters k and j of the trade-off equations, regular inflation cheat neither workers nor firms systematically while as long as productivity is not changing, the distribution of national income is also constant. Finally, the arrows of the diagram indicate that if the economy is disturbed slightly in either direction, it will return to its initial configuration.

The right picture represents a stable short-run demand-limited equilibrium at the intersection between the downward sloping curve $\dot{v} = 0$ and the $\dot{N} = 0$ curve on the branch where it is increasing - for which output is demand-limited. In this situation, price exceeds marginal cost, "but the special kind of market imperfection associated with inadequate effective demand keeps output from rising." (ibid, p. 550) Because there is excess supply of goods, the price level may be falling, but if it is, the money wage is just keeping pace, so that the real wage remains constant. Solow however adds: "Such a situation do in fact seem to arise with the price level actually rising despite the

---

[11] The other point of intersection represents an unstable equilibrium.

existence of what any reasonable man would call excess capacity and unemployment" (Solow 1970, 12)

The inelastic supply curve places "an absolute limit on employment". At any equilibrium in either of the two diagrams, the volume of unemployment is measured by the horizontal distance between the equilibrium level of employment to the vertical line N^S.
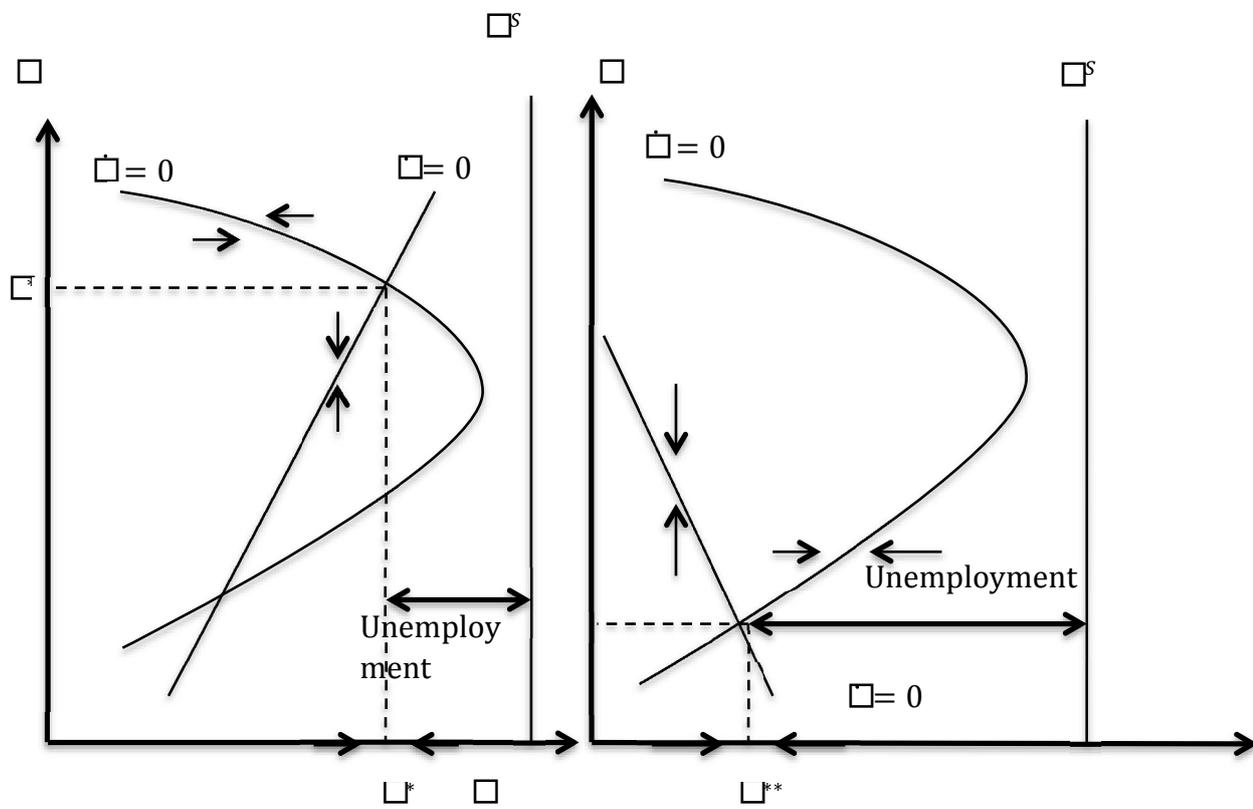
a. Stable supply-constrained equilibrium

b. Stable demand-constrained equlibrium

If, now, both trade-off equations are non-linear - because of variables coefficients j and k, there emerges the possibility to define multiple equilibria so that in "plausible circumstances, the economy may stabilize either to a stable demand- or supply-limited equilibrium. It is precisely by pointing out that configuration - discussed only in passing in his 1968 paper - that Solow questioned Friedman's analysis in his 1970 book.

3. Challenging Friedman's natural rate of unemployment

With changing coefficients k and j of the "trade-off" equations (4) and (5), the constant-v locus may have several upward and downward segments. "Whether it does so or not, it may intertwine with the constant-N locus and intersect any number of times, yielding alternatively stable and unstable equilibria." (Solow and Stiglitz 1968, 548) One may thus get a situation with two stable equilibra, one demand-limited for a low level of employment and one supply-limited for a high level of employment.[12]

The following situation drawn from Solow's 1970 book is particularly interesting.

---

[12] Solow refers here to "plausible circumstances" from a theoretical perspective. It should be noted that no reference is here made to chapter 1 and 2 of Solow 1970 book based on empirical estimations of Phillips curve.

□

$\dot{□}= 0$

□₂

1

E_2

□*   □**   □
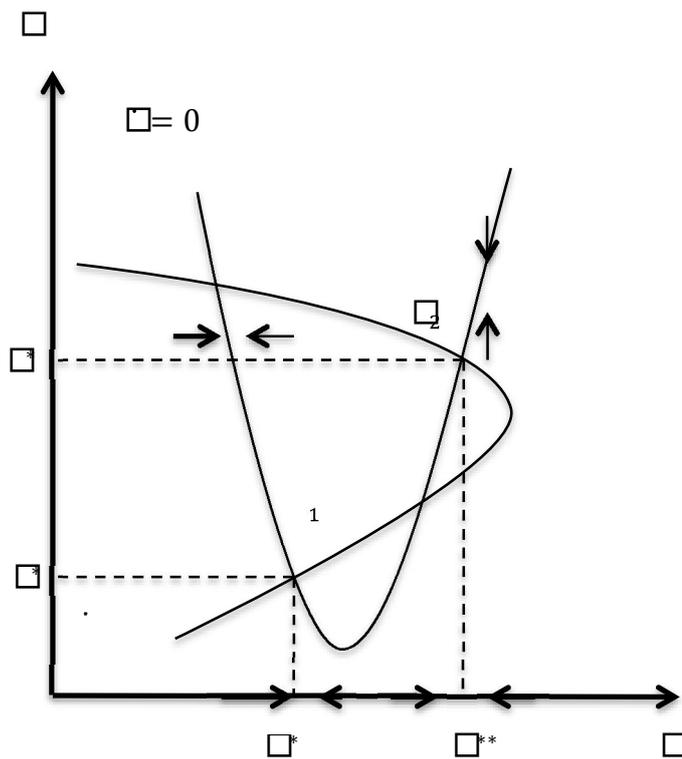
Multiple equilibria

E_1 is a stable equilibrium with excess supply for output. The constant real wage curve intersects the constant employment curve in his rising branch in which aggregate demand consideration predominates. A higher real wage will reduce profit margins but since the limit of output is on the demand side, output will expand. Because the constant real wage curve is decreasing, a higher level of employment will decrease the real wage. As shown by the arrows, in the neighborhood of E_1, employment will necessarily reaches the level N^*.

Symmetrically, E_2 is a stable equilibrium with excess demand. Now, aggregate supply considerations predominate so that a higher real wage will reduce output while real output will be higher if it can be profitably produced at the going real-wage.

However, because the constant real wage curve is now increasing, any rise in employment will come with higher real wage. As a result, in the neighborhood of E_2, employment will necessarily reach the level N^{**}.

The bottom line is that "in plausible circumstances", there might be two levels of income at which the real wage could stabilize.

"In such a case, depending on where it 'begins', the economy may tend in the short run either to an excess-demand equilibrium or to one with excess supply. The interesting possibility emerges that the economy might be jolted out of an underemployment equilibrium and transferred to a new 'initial position' from which it might find its way to an inflationary excess-demand equilibrium, or *vice versa*." (Solow 1970, p. 13)

Assume that the monetary authorities know that the economy may stabilize in E_1 and E_2. If the economy is in E_1 with a level of employment N^*, the issue will clearly be to reach a level of employment N^{**} by an appropriate demand management policy. From E_1, indeed, a shift in aggregate demand will come with a rightwards shift in both the constant-N curve and the constant-v curve. The increase in aggregate demand will accelerate the inflation of the price level. But because the money wage does not initially keep pace, the real wage falls which curbs the rise in employment if the new constant employment curve keeps intersecting the new constant real wage curve in his decreasing branch. However, if the rise in aggregate is strong enough, a new stable equilibrium will be established at the intersection between the increasing constant real wage curve and the decreasing constant employment curves.

It follows that from E_1, the economy may permanently move to an equilibrium similar to E_2 with high level of employment and real wage. Conversely, if from E_2, the

economy is disturbed, it may move into a situation of low real wage and low output similar to E_1.

In both cases, one can see that the equilibrium level is not policy-independent Naturally, the "high" equilibrium with higher employment is better than "low" equilibrium with a low level of employment. Because the model is short-run model, it has nothing to say about the medium to long-run dynamics[13].

Though Solow did not go further in 1970 in his confrontation with Friedman, it is worth here pointing out how much his analysis was an attempt to dismiss the natural rate of unemployment notion and its corollary, the infectiveness of demand policies.

Friedman defined a natural rate of unemployment resulting from market structures and institutional set ups, including factors cited as cost-push forces such as union power. Most importantly, he emphasized the role of inflation expectation in the Phillips curve, and could claim that no trade-off between unemployment and inflation could exist in the long run. As we have shown both ingredients are parts of Solow's analysis. However, by considering the possibility to have multiple stable equilibria, in a configuration in which the economy may be either demand- or supply-constrained, Solow can show that there is no unique stable level of employment associated with non-accelerating rate of unemployment but two or more in which neither firms nor workers are "surprised by inflation". As a result, depending on where the economy is starting, the

---

[13] It was precisely with the aim to address that issue that Solow (2003) attempted to extend the model to the long-run.

use of demand policy may have permanent and positive impact on the level of employment and output!

4. Final remarks: a road not taken

Though Solow felt the need to republish the third chapter of 1970s book, it is important to notice that he did not really follow that line to question Friedman and monetarist economists. One may conjecture that important limitations of his analysis may have decided Solow to abandon this path.

First, as noted earlier, the analysis is devoid of any monetary sector. As a result, the whole analysis implicitly assumes that inflation or deflation associated with any stable equilibrium would have no direct effect on investment. In the context of the 1970s, it was difficult to assume that the money rate of interest would remain in a context of high inflation. Second, and maybe most importantly, Solow's analysis is based on a particular treatment of non-linearities - represented by the coefficients k and j - of the two price and wages equations. The problem is that without a solid theory of these non-linearities in relation to institutions and the formation of expectation, the analysis seems to lack sound foundations. Another possibility would have been to focus on non-linearities in relation to the shape functions $g(\cdot)$ and $h(\cdot)$ of the price and wage equation. This is the path that Solow decided to take from the 1980s in search for medium-Run macroeconomics with the help to Wage Setting-Price Setting models but in a context in which the opposition between demand and supply-limited equilibrium was definitely ruled out.

References

Backhouse, R. E. and Boianovsky, M. 2013, *Transforming Modern Macroeconomics.*, Cambridge University Press.

Friedman, M. 1968, «The Role of Monetary Policy ». *The American Economic Review*, 58, 1, 1–17.

Friedman, M. 1977, «Nobel Lecture: Inflation and Unemployment», *The Journal of Political Economy*, 85, 3, 451–472.

Kaldor, N. 1955, «Alterative Theories of Distribution», *Review of Economic Studies*, 23, 2, 83-100.

Korliras, P. G. and Thorn, R. S. 1979, *Modern Macroeconomics: Major Contributions to Contemporary Thought*, New York : Harper & Row.

Lipsey, R. G. 1960, «The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1882-1957: A Further Analysis», *Economica*, 27, 105, 1-31.

Phillips, A. W. H. 1958, «The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957», *Economica*, 25, 100, 283–299.

Samuelson, P. A., and Solow. R. M. 1960, «Analytical Aspects of Anti-Inflation Policy». *American Economic Review*, 50, 2, 177-94.

Schwarzer, J. A. 2013, «Samuelson and Solow on the Phillips Curve and the "Menu of Choice": A Retrospective», *Œconomia*, 3, 3, 359-389..

Solow, R. M. 1970, *Price Expectations and the Behaviour of the Price Level*. Manchester: Manchester University Press.

Solow, R. 2003, «Stumbling toward the Medium-Run macroeconomics» in *Economics for an Imperfect World. Essays in Honor of Joseph E. Stiglitz*, Arnott. R., Greenwald. B., Kanbur, R. and Nalebuff, B. (eds.), MIT Press.

Solow, R. M., Taylor J. B. and Mankiw, N. G. 2009, «Fifty Years of the Phillips Curve: A Dialog on What We Have Learned» In *Understanding Inflation and the Implications for Monetary Policy: A Phillips Curve Retrospective*. Fuhrer, J., Kodrzycki, Y. K., Little, J. S. and Olivei, G. P. (eds.), Cambridge, MA: The MIT Press.

Solow, R., and J. Stiglitz. 1968, «Output, Employment, and Wages in the Short Run»,

The Quarterly Journal of Economics, 82, 4, 537–60.

Weintraub, S. 1970. «Solow and Stiglitz on Employment and Distribution: A New Romance with an Old Model? », *The Quarterly Journal of Economics*, 84, 1, 144-152.